THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY IN NEW YORK.

THE one hundred and seventieth regular meeting of the Society was held in New York City on Saturday, April 25, 1914, extending through the usual morning and afternoon sessions. The attendance included the following forty-four members:

Mr. E. S. Allen, Dr. F. W. Beal, Mr. R. D. Beetle, Professor W. J. Berry, Professor E. G. Bill, Professor G. D. Birkhoff, Professor Maxime Bôcher, Professor Joseph Bowden, Professor B. H. Camp, Professor F. N. Cole, Dr. G. M. Conwell, Professor J. L. Coolidge, Professor Elizabeth B. Cowley, Professor L. P. Eisenhart, Dr. C. A. Fischer, Professor T. S. Fiske, Professor W. B. Fite, Dr. G. M. Green, Dr. T. H. Gronwall, Professor C. N. Haskins, Professor H. E. Hawkes, Mr. S. A. Joffe, Professor Edward Kasner, Dr. J. K. Lamond, Professor W. R. Longley, Professor James Maclay, Dr. E. J. Miles, Mr. B. E. Mitchell, Dr. H. H. Mitchell, Professor C. L. E. Moore, Dr. R. L. Moore, Dr. F. M. Morgan, Professor Alexander Pell, Dr. Anna J. Pell, Dr. H. W. Reddick, Professor R. G. D. Richardson, Dr. J. E. Rowe, Dr. Clara E. Smith, Professor D. E. Smith, Professor P. F. Smith, Mr. J. M. Stetson, Professor H. W. Tyler, Mr. H. S. Vandiver, Mr. H. E. Webb.

Ex-President Maxime Bôcher occupied the chair, being relieved by Vice-President L. P. Eisenhart, Ex-President T. S. Fiske, and Professor H. W. Tyler. The Council announced the election of the following persons to membership in the Society: Dr. T. H. Brown, Yale University; Dr. Josephine E. Burns, University of Illinois; Professor C. F. Gummer, Queen's University, Kingston, Ontario; Mr. G. M. Hayes, College of the City of New York; Dr. W. L. Miser, University of Minnesota; Professor Maximilian Philip, College of the City of New York; Mr. S. A. Schwarz, College of the City of New York; Professor A. G. Smith, State University of Iowa; Mr. R. A. Wetzel, College of the City of New York. Ten applications for membership in the Society were received.

Professor L. E. Dickson was reelected as member of the

editorial committee of the *Transactions* for a term of three years beginning October, 1914. It was decided to hold the annual meeting of the Society this year in New York City.

Fifteen members gathered at the usual dinner in the evening.

The publication was announced of the Madison Colloquium Lectures of Professors L. E. Dickson and W. F. Osgood. The book has just appeared as volume 4 of the series of Colloquium Lectures.

The following papers were read at this meeting:

(1) Mr. A. R. SCHWEITZER: "An extension of functional equations."

(2) Professor L. P. EISENHART: "Transformations of conjugate systems with equal point invariants."

(3) Dr. H. H. MITCHELL: "The subgroups of the quaternary abelian linear group."

(4) Mr. R. D. BEETLE: "On the complete independence of Schimmack's postulates for the arithmetic mean."

(5) Mr. H. S. VANDIVER: "Extension of the criteria of Wieferich and Mirimanoff in connection with Fermat's last theorem."

(6) Professor E. G. BILL: "Note on the curvature of a regular curve in non-euclidean space."

(7) Mr. S. A. JOFFE: "Triangles whose sides are three consecutive integers and whose area is an integer."

(8) Professor G. D. BIRKHOFF: "The restricted problem of three bodies. Second paper."

(9) Professor A. B. COBLE: "Cremona groups determined by point sets."

(10) Dr. ANNA J. PELL: "Non-homogeneous linear equations in infinitely many unknowns."

(11) Miss L. D. CUMMINGS: "On a method of comparison for triple systems."

(12) Professor C. L. E. MOORE: "A geometry whose element of arc is a linear differential form."

(13) Professor C. L. E. MOORE: "On the centers and radii of curvature of curves traced on a surface in an n-space."

(14) Dr. J. E. ROWE: "Invariants of the rational plane quintic and of rational curves of odd order."

(15) Dr. J. K. LAMOND: "Some applications of a theorem of W. H. Young."

(16) Professor MAXIME BÔCHER: "On a small variation which renders a linear differential system incompatible."

(17) Professor R. C. ARCHIBALD: "Euclid's book on divisions of figures."

(18) Mr. C. E. WILDER: "On the degree of approximation to discontinuous functions by trigonometric sums. Second paper."

(19) Dr. E. J. MILES: "Transversality and orthogonality of space extremals."

(20) Mr. P. R. RIDER: "Broken extremals in space."

(21) Dr. R. L. MOORE: "Linear order in terms of point and limit."

(22) Dr. T. H. GRONWALL: "Triply orthogonal systems containing one family of minimal surfaces."

Mr. Wilder's paper was communicated to the Society by Dr. Dunham Jackson. Mr. Rider was introduced by Dr. Miles. The papers of Mr. Schweitzer, Professor Coble, Miss Cummings, Professor Bôcher, Professor Archibald, Mr. Wilder, and Dr. Miles were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Mr. Schweitzer's paper is concerned with the determination of the properties of the functions satisfying the following functional equations:

$$\psi\{\theta(x)\phi(y) - \theta(y)\phi(x)\} = \sqrt{\psi\theta(x+y)}\sqrt{\psi\theta(x-y)},$$

$$\psi_1\{\theta_1(x)\theta_1(y) - \phi_1(x)\phi_1(y)\} = \sqrt{\psi_1\theta_1(x+y)}\sqrt{\psi_1\theta_1(x-y)}.$$

By putting $\phi(x) = 1 - \theta(x)$ and $\phi_1(x) = 1 - \theta_1(x)$, one obtains functional equations previously discussed by the author.

2. When the cartesian coordinates of a surface S are solutions of an equation of the form

$$\frac{\partial^2\theta}{\partial u\partial v} + \frac{\partial\rho}{\partial v}\frac{\partial\theta}{\partial u} + \frac{\partial\rho}{\partial u}\frac{\partial\theta}{\partial v} = 0,$$

the parametric conjugate system is said to have equal point invariants. Professor Eisenhart shows that when another solution of this equation is known which is linearly independent of the coordinates of S, the coordinates of a second surface S_1, referred to a similar conjugate system, can be

found by quadratures, the surfaee S_1 being such that for the congruence of lines joining corresponding points on S and S_1 the developables meet the latter in the parametric conjugate systems. This result is established by an adaptation of an important theorem due to Moutard. This transformation of S into S_1 is called a transformation K. If two transforms S_1 and S_2 of a surface S are known, a one-parameter family of surfaces S' can be found by quadratures such that each surface S' is in the relation of a transformation K with S_1 and of a different transformation K with S_2; moreover, the corresponding points M, M_1, M_2, M' on four surfaces S, S_1, S_2, S' lie in a plane π, and all the corresponding points M' for surfaces of the family lie on a line through M. This line and the line M_1M_2 meet in the point of contact of the plane π with its envelope. These results are equally true in n-space of reseaux and congruences in the sense adopted by Guichard. In 3-space a transformation K gives rise to four W-congruences in the relation of a theorem of permutability as considered by Bianchi, and conversely four such W-congruences determine four pairs of surfaces in the relations of transformations K. The transformations D_m of isothermic surfaces, studied by Darboux and Bianchi, are the only transformations K for which the two surfaces envelop a two-parameter family of spheres. The equations of the theorem of permutability of the transformations D_m are an immediate consequence of the general equations for transformations K. The results of this paper reveal the fundamental rôle played by the theorem of Moutard in the recent theory of transformations of surfaces.

3. One of the most interesting of the modular groups is the abelian linear group. For the Galois field $GF(p)$ it has been investigated by Jordan, and for the general Galois field $GF(p^n)$ by Dickson. For the case of four variables considerable work has been done on the determination of its subgroups, but the problem seems to have been completely solved only for $n = 1$, $p = 2, 3$. Dickson however has proved that for $n = 1$, $p > 3$ there is no subgroup of index less than $p^3 + p^2 + p + 1$. Dr. Mitchell determines the maximal subgroups in case p is an odd prime and n any integer. He finds that except for $n = 1$, $p = 3$ there is no subgroup of index less than $p^{3n} + p^{2n} + p^n + 1$.

4. In volume 68 of the *Mathematische Annalen*, Schimmack gave a set of axioms for the arithmetic mean, and showed that these axioms were independent in the usual sense. Mr. Beetle proves that Schimmack's axioms form an interesting example of a completely independent* set.

5. If

$$x^p + y^p + z^p = 0 \tag{1}$$

is satisfied in rational integers prime to each other and to the odd prime p, Mr. Vandiver shows that

$$\prod_{i=1}^{m-1}(\alpha^{2i} - t)\sum_{i=1}^{m-1}\frac{(1-\alpha^{2i})f_{p-1}(\alpha^i)}{(1-\alpha^{ip})(\alpha^{2i}-t)} \equiv 0 \pmod{p},$$

where $\alpha = e^{\frac{2\pi\sqrt{-1}}{m}}$ $(m = 2, 3, \cdots, p-1)$,

$$f_k(a) = \sum_{r=1}^{p-1} r^{k-1}a^r,$$

and t is any of the ratios

$$-\frac{x}{y},\quad -\frac{y}{x},\quad -\frac{x}{z},\quad -\frac{z}{x},\quad -\frac{y}{z},\quad -\frac{z}{y}.$$

Letting $m = 5$ and comparing the resulting congruence with a condition given by Mirimanoff, it follows that $5^{p-1} \equiv 1 \pmod{p^2}$ is a criterion for the solution of (1) in rational integers prime to each other and to p.

6. In the two ordinary non-euclidean spaces, two tangents to a regular curve C at the points with parameter values s and $s + \Delta s$ form four angles $\Delta_i\theta$ $(i = 1, 2, 3, 4)$. Of the four limiting values $\lim_{\Delta s=0}\Delta_i\theta/\Delta s$, two are identically zero and two are numerically equal and not identically zero. Professor Bill defines the curvature of C at s to be one of these limiting values which is not identically zero.

7. Assuming the three sides of a triangle, $a = 2n - 1$, $b = 2n$, and $c = 2n + 1$, so that the area is

$$F = n\sqrt{3(n+1)(n-1)},$$

* See E. H. Moore: "Introduction to a form of general analysis," New Haven Mathematical Colloquium, Yale University Press, 1910, p. 82.

Mr. Joffe shows that all the triangles whose sides are three consecutive integers and whose areas are integers can be obtained from the integral solutions of two indeterminate quadratic equations: $y^2 = 3x^2 - 2$ and $y'^2 = 3x'^2 + 1$. He then proves that all the integral roots, x, y, and x', y', of these quadratic equations may be obtained simultaneously by successive addition, and exhibits the results in a table: the proof being based upon two fundamental identities $\Delta x_i = 2\Sigma x_i$ and $y_i = 2x_i - x_{i-1}$, and corresponding identities for x_i' and y_i'.

The sides b_i and b_i' are computed (in two tables) from the formulas $b_i = 2(y_i^2 + 1)$ and $b_i' = 2(2y_i'^2 - 1)$, and it develops that the following relation holds between three consecutive b's:

$$b_i - 14b_{i-1} + b_{i-2} = 0,$$

and that exactly the same relation exists in the b''s; furthermore, that $b_i = 2y'_{2i-1}$, and $b_i' = 2y'_{2i}$. The b's and b''s are connected with each other by the formula $b_i^2 - 2 = b'_{2i-1}$, while $b_i'^2 - 2 = b'_{2i}$.

8. Levi-Civita* has given a notable form of the equations of motion in the restricted problem of three bodies which is free of singularity at collision of the "infinitesimal body" with *one* of the other two bodies. The paper by Professor Birkhoff contains a new form of the equations of motion free of singularity at collision with either of the two bodies, and thus throughout the entire motion. Interesting applications of this result are possible.

A rigorous demonstration of the existence of retrograde periodic orbits, so long as the motion is confined to an oval about one of the bodies, is obtained. Up to the present time all proofs of existence of periodic orbits have dealt exclusively with the case in which one of the bodies is sufficiently small or the constant of Jacobi is sufficiently large. F. R. Moulton has called attention to the superior simplicity of the retrograde orbits and has made interesting conjectures as to their character.†

The very difficult questions connected with the direct

* *Acta Mathematica*, vol. 30 (1906), pp. 305–327.

† Proceedings of the Fifth International Congress of Mathematicians, vol. 2, pp. 182–187.

orbits are considered by Professor Birkhoff, but equally definite results are not obtained.

This paper will appear incorporated with a paper on the same subject presented to the Society at New York at the last annual meeting.

9. In Professor Coble's paper the definition of an invariant of a binary form of order n in terms of the coordinates of the corresponding n points in an S_1 is generalized to apply to a set $P_n^{(k)}$ of n points in an S_k. It appears that a set $P_n^{(k)}$ is associated with a set $P_n^{(n-k-2)}$ in such a way that either set projectively defines the other and that corresponding invariants are proportional. If the projectively distinct sets $P_n^{(k)}$ or their associated sets be mapped upon the points of a space $\Sigma_{k(n-k-2)}$, the permutations of the points within a set determine in Σ a Cremona $G_{n!}^{(k)}$. The groups thus determined by $P_n^{(1)}$ or $P_n^{(n-3)}$ are the cross-ratio groups of Moore. Those determined by $P_n^{(r+1)}$ or $P_n^{(n-3-r)}$, $r > 0$, are of additional interest due to the fact that they are subgroups of more comprehensive Cremona groups. For example $P_6^{(2)}$ determines in Σ_4 a Cremona G_{51840} isomorphic with the group of the lines on a cubic surface; $P_7^{(2)}$ or $P_7^{(3)}$ determines in Σ_6 a Cremona $G_{7!\cdot 288}$ isomorphic with that of the bitangents of a plane quartic; and $P_8^{(2)}$ or $P_8^{(4)}$ determines in Σ_8 a Cremona group isomorphic with that of the tritangent planes of a space sextic of genus 4 on a quadric cone, i. e., a sextic for which an even theta vanishes for the zero argument. In all these cases the group is in direct algebraic relation with the corresponding geometric configuration. The extended Cremona groups determined by sets of points other than those mentioned are of infinite order.

10. Using the property of a limited bilinear form that its value can be obtained by summation by rows or by columns, Mrs. Pell shows that the system of non-homogeneous linear equations

$$\sum_{k=i}^{\infty} a_{ik}x_k = c_i \qquad (i = 1, 2, \cdots, \infty),$$

$a_{ii} \neq 0$ being the only restriction on the coefficients, has a solution $\{\gamma_i\}$ such that $\{\lambda_i\gamma_i\}$ is of finite norm, for every $\{c_i\}$ such that $\{\mu_i c_i\}$ is of finite norm. The constants $\{\lambda_i\}$

and $\{\mu_i\}$ are directly and simply expressed in terms of the coefficients a_{ik}. Extension is made to systems of linear equations reducible to this type.

11. In Miss Cummings's paper three points fundamental in the study of triple-systems are considered: (*a*) A method of comparison applicable to triple-systems of any number of elements; (*b*) an easy process for constructing non-congruent systems; (*c*) a direct means for determining the sets of transitive elements and the group of a system. That the group does not form an abstract mark for systems containing more than 13 elements is established, and several examples are shown of non-congruent systems with the same group. In this paper a system is characterized by its *indices*. Two systems with different sets of indices are shown to be distinct, whether their groups are different or identical. Of the 24 systems discussed at least 12 are apparently new. The known systems of Kirkman, Reiss, Netto, Heffter, and E. H. Moore are examined; and the identity or non-congruency of these systems is exhibited. The groups are determined for the 24 systems.

12. It is evident that there are but two kinds of geometries here possible, viz., (1) arc length an exact differential, (2) arc length not an exact differential. The geometries developed by Phillips and Moore* are examples of each kind. Therefore any geometry whose arc length is linear can be mapped (arc length being preserved) on one or the other of these. Using this map, Professor Moore discusses the geometry on a minimum developable surface.

13. Making use of the well known theorems that the osculating planes of all curves traced on a surface through a point P in a fixed direction t generate a 3-space, and that the locus of this 3-space as t varies is a quadric cone of four dimensions situated in a 5-space, Professor Moore discusses the properties of radii of curvature and of centers of curvature of curves traced through a fixed point on a surface in S_n.

14. In his dissertation (*Transactions*, July, 1911) Dr. Rowe has shown how to interpret certain combinants of two binary

* "An algebra of plane projective geometry," *Proceedings Amer. Academy of Arts and Sciences*, vol. 47 (1912).

quartics (explicitly written out in Salmon's Higher Algebra) as invariants of the rational plane quartic curve. In this paper he shows how these same combinants may be interpreted as invariants of any rational curve of odd order by means of substitutions which are easily derived. This leads to the development of some interesting facts in regard to rational curves. The rational plane quintic is of especial interest because of its self-conjugacy, and the substitutions which lead to eight of its invariants are actually carried out to form a basis of work in the invariant theory of the rational quintic and to illustrate the general process.

15. In a paper, "On the general theory of integration," published in the *Philosophical Transactions* in 1905, W. H. Young stated and proved a theorem which expressed the Lebesgue integral of an upper semi-continuous function as the Riemann integral of a monotone decreasing function over an interval. In a previous paper Dr. Lamond stated a generalization of the above theorem and used it to obtain sufficient conditions for the existence and equality of the multiple and iterated L-integrals of a function over a field, which was not, necessarily, measurable. From his generalization of Young's theorem Dr. Lamond has now shown that the above sufficient conditions are also, in general, necessary, and has also applied this generalization to the problem of the continuity of an L-integral with respect to a parameter.

16. If the homogeneous linear differential equation of the nth order, $L(u) = 0$, has k linearly independent solutions which satisfy a certain system of homogeneous linear boundary conditions, $U_i(u) = 0$ $(i = 1, 2, \cdots, n)$, the main result proved in Professor Bôcher's paper states that, however small the positive constant ϵ may be, a continuous function $g(x)$ exists such that $0 < g(x) < \epsilon$ and such that the differential equation $L(u) = g(x)u$ has no solution except zero which satisfies the boundary conditions in question.

17. Euclid was the author of not less than nine works. We have approximately complete texts, all carefully edited, of four of these: (1) the Elements, (2) the Data, (3) the Optics, (4) the Phaenomena. In the case of (5) the Pseudaria, (6) the Surface Loci, (7) the Conics, our fragmentary knowl-

edge, derived wholly from Greek sources, makes conjecture as to their content of the vaguest nature. On (8) the Porisms, Pappus gives extended comment. To (9) the Book on Divisions of Figures, Proclus alone among Greeks makes slight explanatory reference. But in an Arab MS. translated by Woepcke into French (*Journal Asiatique*, 1851) we have not only the enunciation of all 36 of the propositions of the Book, but also the proofs of four of them. Professor Archibald has endeavored to restore the remaining proofs in euclidean manner, to edit the whole with suitable commentary and historical introduction, which, among other things, takes due account of various Dee and Cotton MSS. attributed to Muhammed Bagdedinus as author. The restoration which is based on Leonardo's Geometria Practicæ (written in 1220 and published by Boncompagni in 1862) will soon be published in England, in book form.

18. This paper is a continuation of the one presented to the Society by Mr. Wilder in February, 1913. It contains several theorems of a more general character than those of the previous paper. The following are typical:

Corresponding to any finite integrable function $f(x)$, of period 2π, there is for every positive integral value of n a trigonometric sum of order n at most which approximates to $f(x)$ at any point where its derived numbers are all finite, with an error not exceeding $1/n$ multiplied by a quantity independent of n.

Any finite integrable function $f(x, y)$, of period 2π in each variable, can be approximated to by an arithmetic mean of partial sums of its Fourier's expansion to terms of order $m-1$ in x and $n-1$ in y, at any point having a neighborhood of radius $\delta < \pi$ in which $f(x, y)$ satisfies the Lipschitz condition

$$|f(x_2, y_2) - f(x_1, y_1)| \leqq \lambda \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

with an error not exceeding

$$\left(\frac{\log m}{m} + \frac{\log n}{n}\right)\left(c_1\lambda + c_2 \frac{\nu}{\delta^2}\right),$$

where c_1 and c_2 are absolute constants, and ν is the difference between the upper and lower limits of $f(x, y)$.

Some of the results can be interpreted so as to give infor-

mation concerning the accuracy of approximation by means of polynomials.

19. In Dr. Miles's paper the general form of the integrand function is first obtained when the extremals which are transversal to a given surface are also perpendicular to that surface. Then a general relation between the direction of the extremal and the transversal surface is assumed and conditions given under which the integrand function can be determined.

20. Caratheodory and Bolza have treated the case of broken extremals in the plane. Mr. Rider's paper considers the case of broken extremals in space. The Weierstrass-Erdmann corner-condition is derived, and a corner-surface obtained. The analogue of Caratheodory's Ω-function is then found, and the Jacobi necessary condition for an extremum derived, as are also sufficient conditions.

21. Dr. Moore adds the following four axioms to a set of three fundamental axioms formulated by F. Riesz,* and thereby secures a set of seven independent axioms, in terms of point and limit, which is a sufficient basis for a definition of a linear order satisfying Hilbert's axioms of group II.†

In the following, S denotes the set of all points under consideration. A set of points is said to be connected if, however it be divided into two complementary subsets, one of them contains a limit point of the other.

Axiom 1. S is connected.

Axiom 2. If P is a point of S, then $S - P$ is composed of two connected subsets neither of which contains a limit point of the other.‡

Axiom 3. There do not exist three rays such that no two of them have a point in common.

Axiom 4. There exist at least two points.

22. In this paper, Professor Gronwall shows that there exist no triply orthogonal systems containing one family of minimal surfaces, except those already known, where the minimal surfaces are either planes or catenoids of revolution.

F. N. COLE, *Secretary.*

* Rome Congress, 1908.

† Hilbert, The Foundations of Geometry, Townsend's translation.

‡ Such subsets are called rays.

THE TWENTY-FIFTH REGULAR MEETING OF THE SAN FRANCISCO SECTION.

THE twenty-fifth regular meeting of the San Francisco Section of the Society was held at the University of Washington on May 22, 1914. Thirty-two persons were present, including the following members of the Society:

Professor R. E. Allardice, Professor W. A. Bratton, Dr. Thomas Buck, Professor A. F. Carpenter, Professor E. E. De Cou, Professor G. I. Gavett, Professor F. L. Griffin, Professor F. W. Hanawalt, Professor H. B. Leonard, Professor W. A. Manning, Professor R. E. Moritz, Professor F. M. Morrison, Dr. L. I. Neikirk, Dr. L. L. Smail, Professor W. M. Smith, Professor R. M. Winger.

Professor Manning, chairman of the section, presided. The members present lunched together between sessions at the Faculty Club, and were entertained in the evening at the home of Professor Moritz.

The following papers were presented at this meeting:

(1) Professor W. M. SMITH: "An associated curve."

(2) Dr. L. I. NEIKIRK: "The functional variable."

(3) Professor F. L. GRIFFIN: "Note on a measure of the variability of percentages in unequal samples recently proposed by H. B. Frost."

(4) Professor F. L. GRIFFIN: "An experiment in mathematical pedagogy."

(5) Dr. L. E. WEAR: "Self-dual rational quartics."

(6) Professor F. M. MORRISON: "On the relation between some important notions of projective and metric differential geometry."

(7) Professor A. F. CARPENTER: "Ruled surfaces with plane flecnode curves" (preliminary report).

(8) Dr. E. T. BELL: "Tables for the multiplication of four double theta functions."

(9) Dr. E. T. BELL: "An arithmetical theory of certain numerical functions."

(10) Dr. L. L. SMAIL: "Some theorems on triple limits."

(11) Dr. L. L. SMAIL: "A new general method for the summation of divergent series."

(12) Professor R. E. MORITZ: "On the general theory of cyclic-harmonic curves."

(13) Professor R. E. MORITZ: "The cyclo-harmonograph: a new mechanism for tracing curves whose equation is $\rho = a \cos p\theta/q + k$."

(14) Professor R. M. WINGER: "The sextic invariant of the Hesse group" (preliminary report).

Dr. Wear was introduced by Professor Moritz, and Dr. Bell by Dr. Neikirk. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

2. The study of integral equations has in recent years led to the development of a large body of theory called functional calculus.* One part of this is the theory of the functions of a line. Volterra† calls any quantity which depends for its value on the arc of a curve as a whole a function of the line.‡ He defines the partial derivative of the function at a point and derives a formula for the differential (variation). He extends the results to functions of more than one line. Following Volterra there have been several papers published along similar lines. In the present paper Dr. Neikirk gives a new definition of the function of a functional variable and also a new definition of the partial derivative at a point. He gets from this definition a mean value theorem and a Taylor's expansion. The mean value theorem gives, among other things, Euler's differential equation as a necessary condition in the calculus of variations. An example of the derivative is the following: the partial derivative of the length of the curve at a point is the negative of the curvature at the point, except for the end points, where it is infinite. A number of other results are given in the paper.

3. In studying the variation of percentages in a series of given samples, it is customary to compare the actual standard deviation for the given series with a theoretical limiting value, viz., $\sqrt{pq/h}$, where p = theoretical percentage for an indefinitely large number of cases, $q = 1 - p$, and h = harmonic mean size of the samples. Mr. Frost proposes to weight each sample with the number of individuals it contains,

* See Winter, "Les principes du calcul fonctionnel," *Revue de Métaphysique et de Morale*, July, 1913.

† *Rendiconti della R. Accad. Dei Lincei*, vol. 3.

‡ See Hadamard, Leçons sur le Calcul des Variations, pp. 281–303.

and thus find a sort of generalized standard deviation, whose theoretical limiting value is $\sqrt{pq/\bar{n}}$, where $\bar{n}$ = arithmetical mean size of sample. Professor Griffin's note merely proves the mathematical validity of the process. It will form part of a paper intended for joint publication in the *American Naturalist.*

4. In this paper Professor Griffin describes a course taught for the past three years under the title of "An introduction to mathematical analysis," which has been evolved to meet these generally recognized needs: (1) for an earlier introduction of calculus, especially integral calculus; (2) for a closer correlation of the calculus with the preliminary subjects, and a consequent postponement of unessential parts of those subjects; and (3) for a fuller exposition of the actual uses of each topic at the time it is studied. A detailed outline is presented, showing how it is possible for students to acquire in a single year a considerable familiarity with graphical methods, calculus, trigonometry, college algebra, and analytic geometry,—in fact, to use elementary integral calculus in their first semester with a fair degree of facility and understanding. As the course presupposes nothing beyond elementary algebra and geometry, its subject matter is within the reach of students in the fourth year of preparatory schools. Some possible modifications and gains in later courses are also described.

5. A necessary condition for the self-duality of a plane curve is that the class of the curve be equal to the order, i. e., $n = n(n-1) - 2d - 3c$. For the quartic curve there are two solutions, viz.,

$$d = 1, \quad c = 2; \quad d = 4, \quad c = 0,$$

which correspond respectively to the limaçon and the degenerate case of two conics. Dr. Wear finds that the former is invariant under a G_4 consisting of identity, a reflexion, and two polarities. The latter are invariant under a G_8 consisting of four collineations and four correlations. By specializing the two conics, groups of orders 16 and 48 may be obtained.

6. In his papers on the "Projective differential geometry of curved surfaces," *Transactions,* volumes 8, 9, and 10,

Professor E. J. Wilczynski has shown that the projective differential geometry of a surface may be based upon the consideration of a completely integrable system of two linear partial differential equations of the second order. In his study of the properties of a surface in the neighborhood of any one of its points, he has systematically used a certain semicovariant tetrahedron of reference. If the asymptotic lines are taken as the parametric lines, the differential equations used by Professor Wilczynski take exactly the same form as the familiar Gauss equations, and we can express directly the coefficients of the differential equations in terms of the Christoffel symbols used in the metrical theory of surfaces. We are thus able to change from the homogenous coordinate system to any suitable cartesian system.

Professor Morrison develops the transformations from the homogenous system to the special cartesian system consisting of the lines of curvature tangents, and the normal to the surface, and makes a study of some of the metrical properties of certain geometrical configurations associated with a point on a surface which have been defined and studied from a projective point of view by Professor Wilczynski. A special study is made of the metrical properties of the osculating linear complexes of the asymptotic curves and the pencil of complexes formed from these two. By a specialization upon the two directrices appearing in this pencil certain special points on a surface and special types of surfaces are defined and a study is made of these.

7. In this paper, Professor Carpenter sets up, in invariant form, the conditions for plane flecnode curves, and applies these conditions to the case of anharmonic curves and in particular to the case of conics.

8. There are 121 cases of the formula of H. J. S. Smith for the multiplication of four double theta functions (*Proceedings of the London Mathematical Society*, volume 10, pages 87–91; also Krazer, Lehrbuch der Thetafunktionen, Chapter VII; § 10). In using the double thetas, it was found convenient to have a compact form from which all of the biquadratic and other relations could be read off; it is the purpose of Dr. Bell's tables to supply this. Table I is a square matrix of the integers 0 to 15 with proper signs; Table II is a square matrix

of the symbols (μ, μ', ν, ν') equivalent respectively to Hermite's $(\substack{\mu' \\ \mu}; \substack{\nu' \\ \nu})$. It is explained with the tables how any one of the 121 quadruple products may be found by a single reference to each of the tables. For the triple thetas it is shown that there is a corresponding scheme in three dimensions; to Table I corresponds a cube, three of whose faces meeting in 0 are identical with Table I.

9. In this paper Dr. Bell continues a paper on "Numerical functions" presented to the Society at the New York meeting, October, 1912. The concept of prime functions here introduced simplifies the theory, whose objects are (1) to exhibit in concise form the interrelations arising in arithmetic between functions $f(n), f_1(n), \cdots$ which exist only when n is a positive integer, and are such that $f(mn) = f(m)f(n)$ when, and in general only when, m, n are relatively prime, $f(\ )$ being any one of the functions; (2) to construct a theory of the properties of these functions that shall be isomorphic to the theory of numbers. By definition, $f(n)$ is a unit function if for $n > 1$, $f(n) = 0$, and for $n = 1$, $f(n) = 1$. If n be factored in all possible ways into r factors, $n_1, n_2, \cdots, n_r$, and the sum of all the corresponding products $f_1(n_1)f_2(n_2) \cdots f_r(n_r)$ be formed, the result being regarded as a function of n, say $f(n)$, then $f(n)$ is defined to be the ideal product of $f_1(n), f_2(n), \cdots, f_r(n)$ (note that in the ideal product the arguments each $= n$). Examples of $f(\ )$ functions in use are the totient, the sum and number of divisors of an integer, $\mu(n)$ (Mertens), $\lambda(n)$ (Dirichlet); in all about 30. It is shown (1) that these are members of a class of functions denumerably infinite in number, (2) that any of these ∞ functions may be resolved into its ideal prime function factors in one way only, a prime function being an $f(\ )$ whose only factors are $f(\ )$ and unit functions; (3) that there are ∞ unit functions; (4) that any theorem in arithmetic regarding divisibility has a unique correspondent in this theory, as e. g., the number of prime functions is infinite; quotients are unique, etc. The theory of ideal addition for $f(\ )$ functions is constructed and shown to be isomorphic to addition for numbers; as also the theories of residuation, congruences, etc. These are too lengthy for abstraction.

10. Dr. Smail's first paper gives conditions under which certain triple limits exist, and gives relations between simultaneous and repeated triple limits.

11. In his Columbia dissertation (1913) (presented to the Society, April 26, 1913), Dr. Smail gave four general methods for the summation of divergent series. In the present paper, he gives a general method of summation which includes three of the four methods just mentioned. The general properties of this method are studied, and applied to the various special methods included in this general method. The known methods of summation of Cesàro, Riesz, Le Roy, Borel, Euler's power series method, and a method related to the Cesàro-Riesz methods of Hardy and Chapman, all appear as special cases under the general method here discussed.

12. Any curve whose equation in polar coordinates is $\rho = a \cos p\theta/q + k$, or whose equation may be reduced to this form by a transformation of coordinates, may be generated by a composition of a simple harmonic with a uniform circular motion. For this reason Professor Moritz designates all such curves by the term cyclic-harmonic. This class of curves evidently embraces an infinite number of species, only a few of which have received any systematic consideration. The present paper is the first of several papers dealing with the general properties of the whole class of curves. The topics considered in the first paper are: (1) the cartesian equation of the various species, (2) the number of species having the degree n, (3) the number and location of the axes of symmetry, (4) the number of polar maxima and minima and their location, (5) the number and distribution of double points.

13. The mechanism exhibited by Professor Moritz combines a simple harmonic motion, $\rho = c \cos pt + k$, with a uniform circular motion, $\theta = qt$. A crank-arm of length a imparts simple harmonic motion to a tracing pencil or pen which by means of a sliding clamp may be adjusted to different values of k. The paper on which the curve is to be drawn is fastened to a revolving disk. A train of gears enables p and q to assume separately all integral values from 1 to 10. The mechanism accomplishes thus the continuous description of 63 different species of curves. For each species a and k may be given all values $|a| > 3$ inches, $|k| > 5$ inches, subject only to the restriction $|a| + |k| < 6$ inches.

14. The complete system of invariants of the Hesse ternary

collineation group G_{216} was given by Maschke, *Mathematische Annalen*, volume 33, page 317. Of the five fundamental forms one is a proper sextic curve. Naturally this curve is closely associated with the syzygetic period of cubic curves which the group leaves invariant. Professor Winger discusses the sextic by means of its group property and points out some interesting relations to the system of cubics.

THOMAS BUCK,
Secretary of the Section.

THE RATIO OF THE ARC TO THE CHORD OF AN ANALYTIC CURVE NEED NOT APPROACH UNITY.

BY PROFESSOR EDWARD KASNER.

(Read before the American Mathematical Society, September 9, 1913.)

"IF P is a fixed point on a curve and Q is a point which approaches P along the curve, the limit of the ratio of the arc PQ to the chord PQ is unity." While this statement is frequently made without reservation, it is easy, as in most analogous statements, to construct exceptions in the domain of real functions: by making the curve sufficiently crinkly the limit may become say two, or any assigned number greater than unity.

The object of this note, however, is to point out the necessity for reservation even in the domain of (complex) analytic curves. The limit may then be *less* than unity. For example, in the imaginary parabola

$$y = ix + x^2$$

the value of the limit in question, at the origin, is *not one, but about* .94. The exact value is easily found to be $\frac{2}{3}\sqrt{2}$.

Of course all such exceptions will be imaginary. Thus for a real non-circular ellipse the limit is obviously unity at each of the ∞^1 real points; but of the ∞^2 imaginary points of the ellipse, there are four points at which the limit takes the value .94 +. These are the points at which the tangent is a minimal (or isotropic) line. Thus if we explore *all* the points of the ellipse,

in the complex domain, the limit takes the usual value at all points except four points where the value is .94. The limit is therefore a discontinuous function of the point of the curve.

Regular Elements or Curves in the Plane.

The general theory is very simple. Consider first any analytic curve in the plane in the neighborhood of a point P at which it is regular. The equation of the curve referred to rectangular axes through P can be taken in the form

$$y = c_1x + c_2x^2 + c_3x^3 + \cdots, \tag{1}$$

where the coefficients c are complex numbers.

The chord γ from the point $P(0, 0)$ to the neighboring point $Q(x, y)$ is

$$\begin{aligned}\gamma &= \sqrt{x^2 + y^2} \\ &= x\sqrt{1 + c_1^2 + 2c_1c_2x + (2c_1c_3 + c_2^2)x^2 + \cdots}.\end{aligned} \tag{2}$$

The length of the arc is

$$\begin{aligned}\alpha &= \int_0^x \sqrt{1 + y'^2}\,dx \\ &= \int_0^x \sqrt{1 + c_1^2 + 4c_1c_2x + (6c_1c_3 + 4c_2^2)x^2 + \cdots}\,dx.\end{aligned} \tag{3}$$

If the term $1 + c_1^2$ does not vanish, that is, if the slope c_1 does not equal $\pm i$, then γ and α can be developed into integral power series starting with the first power of x, the coefficient of x in both cases being $\sqrt{1 + c_1^2}$. Letting $x \doteq 0$, we thus have the usual result

$$L \equiv \lim \alpha/\gamma = 1.$$

If, on the other hand, $1 + c_1^2$ does vanish, the developments of γ and α are series in fractional powers of x. Assuming that c_2 does not vanish, the leading term in γ is then

$$\sqrt{2c_1c_2}x^{\frac{3}{2}},$$

and that in α is

$$\int_0^x \sqrt{4c_1c_2x}\,dx = \tfrac{2}{3}\sqrt{4c_1c_2}x^{\frac{3}{2}},$$

where c_1, of course, is $\pm i$. Then our limit becomes

$$L = \tfrac{2}{3}\sqrt{2} = .94+.$$

If however $c_2 = 0$, in addition of course to $1 + c_1^2 = 0$, while $c_3 \neq 0$, the leading terms in (2) and (3) are

$$\sqrt{2c_1c_3}x^{\frac{1}{2}}$$

and

$$\int \sqrt{6c_1c_3}x^{\frac{1}{2}}dx = \tfrac{1}{2}\sqrt{6c_1c_3}x^{\frac{3}{2}},$$

so that

$$L = \tfrac{1}{2}\sqrt{3} = .86+.$$

In the most general exceptional curve, we have say $c_1 = \pm i$, $c_2 = 0$, $c_3 = 0$, $\cdots$, $c_{k-1} = 0$, $c_k \neq 0$, so that the equation of the curve is of the form

$$y = \pm ix + c_kx^k + c_{k+1}x^{k+1} + \cdots \qquad (c_k \neq 0). \tag{4}$$

The curve then has contact of order $k - 1$ with its minimal tangent. We find then

$$\gamma = \sqrt{2c_1c_k}x^{\frac{k+1}{2}} + \cdots, \tag{5}$$

$$\alpha = \frac{2}{k+1}\sqrt{2kc_1c_k}x^{\frac{k+1}{2}} + \cdots, \tag{6}$$

so that

$$L = \lim \frac{\text{arc}}{\text{chord}} = \frac{2\sqrt{k}}{k+1}. \tag{7}$$

THEOREM I. *The limit of the ratio of the arc to the chord at a regular point of an analytic curve is unity provided the tangent at that point is not a minimal line. In the latter event (that is, when the slope is $\pm i$) the limit is given by (7), where $k - 1$ denotes the order of contact of the curve with the minimal tangent.**

In the trivial case where the given curve is a minimal line (instead of merely touching a minimal line), the limit does not exist.

Omitting this trivial case, we see that at every point, real

* That is, k is the number of consecutive points common to the curve and the tangent. In this form we observe that (7) remains valid even when $k = 1$, for then $L = 1$.

or imaginary, the limit exists and is always real. *If it is not one, it is at most* .94 +; if it is less than this, it is at most .86 + ···. Thus the limit value L is always a number selected from the denumerable set

$$\frac{2\sqrt{k}}{k+1} \qquad (k = 1, 2, 3, \cdots),$$

the first few terms being, to two decimal places,

(8) 1.00, .94, .86, .80, .74, ···.

Most of these numbers are irrational, but some are rational. Thus in the case of third order contact with the minimal line, that is, $k = 4$, we have $L = 4/5$.

Notice that in all the exceptional cases L is *less* than unity, that is, the arc becomes less, in absolute value, than the chord. This is as we should expect. For the chord ultimately diverges more from the minimal tangent than does the arc, and along a minimal line distances are zero.

The sequence (8) has zero for its limit. That is, if k is made larger and larger, so that the curve has higher and higher contact with the minimal tangent, the value of L approaches zero. But, as observed before, if we actually let the curve reduce to a minimal line, the limit L ceases to exist. Therefore L cannot be made equal to zero; this number is not contained in the sequence (8); but L can be made smaller than any assigned number.

*Irregular Elements.**

Consider next *irregular* analytic arcs, that is, curves which, in a neighborhood of the given point P, taken as origin of rectangular axes, cannot be represented (for any choice of axes) by setting y equal to a series in integral powers of x, but can be represented by a series with fractional exponents, say a series arranged in powers of the pth root of x. If the slope of the curve at P is not $\pm i$, the usual value $L = 1$ is obtained. Otherwise we have the following

THEOREM II. *If the analytic curve is irregular at the point P, and if the tangent line is minimal, the equation of the curve,*

* We are dealing always with an *analytic element*, that is, an analytic curve at a *given* point. We apply the adjective irregular either to the point or the curve or (preferably) the element. Cf. the usage of Study, § 6.

referred to P as origin, being of the form

$$y = \pm ix + c_{p+k-1}x^{\frac{p+k-1}{p}} + c_{p+k}x^{\frac{p+k}{p}} + \cdots, \tag{9}$$

the limit of the ratio of the arc to the chord is

$$L = \frac{2p}{2p+k-1}\sqrt{\frac{p+k-1}{p}}. \tag{10}$$

For the regular curves we have, of course, $p = 1$, so that this general formula (10) reduces to (7). The range of possible values of L for the irregular arcs is greater, since the result (10) depends on the two integers p and k, characterizing the type of irregular (singular) point and the contact with the minimal tangent. The set of numbers (10) is dense everywhere between 1 and 0, while set (7) is dense only at 0. The value of the limit L may, in particular, now lie between 1 and .94. Thus for an ordinary cusp with minimal tangent, that is, $p = 2$, $k = 2$, we have

$$L = \tfrac{4}{5}\sqrt{\tfrac{3}{2}} = .96+.$$

If p is fixed and k is made large, L becomes nearly 0; while if k is fixed and p made large, L becomes nearly 1.

We observe that L^2 in (10) is always rational and not greater than one. Not every proper fraction, however, can be a value of L^2. The necessary and sufficient condition is that $1 - L^2$ shall be the square of a proper fraction.

A Real Representation of the Results.

Is it possible to bring the exceptional cases we have found within the realm of intuition instead of mere calculation? Can we picture, for example, the limiting ratio .94? One plan would be to use Study's real representation of analytic curves by means of ∞^2 point pairs. The interpretation of distance and arc is then of course real but quite complicated.* We therefore use an indirect plan which is more convenient for our particular purpose.

We pass from the original (x, y) plane to a (u, v) plane by the imaginary affine correspondence

$$u = x + iy, \qquad v = x - iy. \tag{11}$$

* See Study, Vorlesung über ausgewählte Gegenstände der Geometrie, Erstes Heft (1911), § 2.

The element of arc in the (x, y) plane,

$$ds = \sqrt{dx^2 + dy^2}, \tag{12}$$

then becomes

$$d\sigma = \sqrt{du\,dv}. \tag{13}$$

Thus the distance between two points P and Q in the first plane becomes the "pseudo-distance" $P'Q'$, defined *not* as the length of the segment $P'Q'$, but as the side of a square having the same area as the rectangle constructed with $P'Q'$ as diagonal, the sides being parallel to the u, v axes.

The pseudo-length of an *arc* running from P' to Q' is of course obtained by inserting many intermediate points and constructing on the sides of the inscribed broken line little rectangles having those sides as diagonals, then adding, *not* the areas of these rectangles (that would give merely zero in the limit) but the square roots of the areas. The pseudo-length of the *chord* $P'Q'$ is obtained from the single big rectangle constructed for the diagonal $P'Q'$.

With these definitions, we readily find that the ratio of the pseudo-arc to the pseudo-chord approaches unity when Q' approaches P', *except* when the tangent line at P' is horizontal or vertical. In fact the minimal lines in the (x, y) plane are converted by our representation (11) into the lines parallel to the u, v axes. If we take a real curve in the (u, v) plane touching the horizontal line at P', the pseudo-arc $P'Q'$ will ultimately become smaller than the pseudo-chord $P'Q'$. If the contact is of the first order, the limiting value of the ratio is $.94+$, *a result that could be verified by an accurate draughtsman.* The other values of L, corresponding to higher contact and irregular points according to formulas (7) and (10), would be obtained with increasing experimental difficulties.

Space Curves.

So far we have been considering only plane curves. For analytic curves in space (of three or more dimensions) the results are quite different. The limit of the arc to the chord is of course unity for all real curves; and also for all complex curves provided the tangent line at the given point is not minimal. For the exceptional curves, however, the range of possibilities is much greater than in the plane. In the plane curve, the limit is always real. For space curves it may be

imaginary. In the plane, when the limit is not unity, the arc ultimately becomes less in absolute value than the chord. In space it may become either less or greater. *While the set of possible values for L in the plane is denumerable, it constitutes, in space, a continuum.* This is true even if we confine ourselves to regular analytic curves.

To justify these statements, we need not give a complete classification, but may confine ourselves to the special category of curves

$$y = a_1x + a_2x^2 + a_3x^3 + \cdots, \qquad z = b_1x + b_2x^2 + b_3x^3 + \cdots$$

defined by the relations

$$1 + a_1^2 + b_1^2 = 0, \qquad a_1a_2 + b_1b_2 = 0,$$

$$(a_2^2 + b_2^2)(a_1a_3 + b_1b_3) \neq 0.$$

By a simple calculation of the leading terms in the developments for arc and chord we find for such a curve

$$L = \sqrt{\frac{3\lambda + 2}{4\lambda + 2}},$$

where

$$\lambda = \frac{a_1a_3 + b_1b_3}{a_2^2 + b_2^2}.$$

Here L does not depend merely upon certain integers (arithmetic invariants, order of contact), as was the case in the plane, but upon the actual coefficients in the equations of the curve, that is, upon differential invariants. Since λ may take any complex value, the same is true for L.

THEOREM III. *Regular analytic curves in space can be constructed for which, at a given point, L shall have any assigned real or imaginary value.*

Relation to Conformal Geometry.

It is worth while pointing out that the quantity L defined for any point of an analytic curve is important not only in metric geometry, based on the group of displacements, but in the more general conformal geometry, based on the group of all conformal transformations, a group which is infinite in the plane and finite in space. This follows from the fact that the value of L depends on the relation of the given curve to

minimal lines and curves. *In fact the conformal group may be defined as consisting of all point transformations for which L is invariant.* In particular it is found that two regular elements in the plane are conformally equivalent when and only when they agree with respect to L.

This is not true of irregular arcs. The complete discussion of the conformal invariants of irregular analytic curves, or elements, will be given in another paper. The main result is that every such curve has differential invariants (certainly more than one, and probably an infinite number) except in the type represented by an ordinary cusp with non-minimal tangent. This type can be reduced formally to the normal form $y = x^{\frac{3}{2}}$, and therefore has no invariant. Furthermore, any such analytic cusp can be transformed into any other by a *unique* conformal transformation.

The power series defining an analytic element may be real or imaginary, integral or fractional. If, furthermore, we allow the series to be divergent as well as convergent, we have also a classification of elements as divergent or convergent. The concept of *divergent* differential element of infinite order thus introduced promises to lead to greater simplicity and generality in the differential geometry of curves and surfaces. The resulting "geometry of divergent power series" will be treated elsewhere.

COLUMBIA UNIVERSITY,
NEW YORK.

A MERSENNE PRIME.

By Lucas's theorem (*American Journal of Mathematics*, volume 1, page 305) a Mersenne number $n = 2^{4q+3} - 1$ with $4q + 3$ a prime and $8q + 7$ composite is a prime number if the first term of the series 3, 7, 47, 2207, . . . divisible by n lies between the $(2q + 1)$th and the $(4q + 2)$th. My computations show that $2^{107} - 1$ divides the 106th term of the series and is therefore a prime number.

R. E. POWERS.

DENVER, COLO.

OSGOOD'S THEORY OF FUNCTIONS.

Lehrbuch der Funktionentheorie. Von DR. W. F. OSGOOD. Erster Band mit 158 Figuren. Zweite Auflage. (Bd. XX: 1, B. G. Teubner's Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften.) B. G. Teubner, 1912. 8vo. xii + 766 pp.

OSGOOD'S Theory of Functions is a monument of American scholarship. Comparatively few mathematical works of broad, *comprehensive* character have as yet been written by Americans. While there is a superabundance of good or indifferent texts in English on elementary topics such as analytic geometry and elementary calculus, we lack a sufficiency on more advanced subjects. The appearance of a work such as this, realizing the highest standards for breadth and accuracy of scholarship, is an event. It is therefore to be soberly regretted that it has not been published in the English tongue. Written by a man of English name and of true American stock, it appears in the language of that country which needs it least. Yet while I voice a widely expressed regret that Osgood's treatise is not written in English, it must, on the other hand, be recognized that mathematics is international; further, that the inspiration and spirit of the work is in good part German.

Inasmuch as no previous review has appeared in the BULLETIN,* the task of the reviewer should be a double one, not merely to consider the changes occurring in the new edition but also to survey the work as a whole, even though the characteristics and essential features are by this time well known to many readers. The attempt to do this, however, brings despair to the reviewer because of the many-sidedness of the work and its exceeding richness of content. It is amazing what a wealth of ideas and detail is packed into a single volume. The judiciousness of selection and thorough assimilation of material combine to make the work par excellence the most authoritative treatise upon the theory of analytic functions.

I.

The first edition appeared in 1907 and was divided into three Abschnitte. The first of these collects the "Theorems and

* See, however, a translation of the preface by H. S. White, vol. 13, p. 398.

methods of the function theory of real variables" and the concepts and results of the Mengenlehre which are indispensable for a thorough and modern treatment of analytic functions. In so doing the author implicitly recognizes—what has in truth become increasingly apparent with mathematical progress—that a course in real analysis should logically precede a study of the function theory of the complex variable, unless the two theories are interwoven as in French treatises on analysis. This is nearly, if not quite, imperative when the Riemann-Cauchy standpoint is adopted. Though Osgood departs outwardly from the French plan in organizing and separating the function theories of the real and of the complex variable, the dependence of the latter upon the former is made so intimate as to produce essentially the same effect as does their interweaving. After the way has been cleared in Abschnitt I, the elements and principal working theorems of the theory of analytic functions are developed in Abschnitt II with great rapidity. Abschnitt III treats of the applications, which are given in successive chapters on elliptic functions, Reihen- und Produktenentwickelungen, the elementary functions, and logarithmic potential.

In the new edition there is an expansion from the previous 642 pages to 766 pages. The last chapter of Abschnitt III has now been split off and amplified into a fourth Abschnitt containing two chapters, which are devoted to logarithmic potential and to conformal representation and the uniformization of analytic functions respectively. A complete treatment of the last subject constitutes the main addition to the first edition and accounts approximately for a third of the increase in the size of the volume. Of this addition I shall speak further on.

A second change of much importance is the insertion of numerous footnotes concerning the authorship of the theorems and results, their origin, place of publication, etc. The compilation of these footnotes must have been laborious, but they are invaluable and greatly enhance the usefulness of the work, especially for the teacher and investigator.

Beside these major changes numberless minor changes are made from page to page in the new edition, in consequence of which the revision may be called an extensive one, though with practically no change of plan. Not infrequently extensions of theorems are added which result from placing less

restriction upon the hypotheses. Some of the more striking changes in their order of occurrence are as follows.

At the close of chapter 1 the familiar Heine-Borel theorem is appended under the heading: Ein allgemeines Theorem. The author's conception of it is interesting. To quote his words, when in the vicinity of every point of a closed interval a certain property has been established and the continuation of the proof consists in the application of the method of "Einschachtelung der Intervalle," the last part of the proof can be formulated in the aforesaid theorem. The appropriateness of the usual designation of the theorem is not discussed. Also, so far as I have observed, no use is anywhere made of the theorem. This may be regretted, since only through examples will the student gain an idea of its mode of application and of its frequent great convenience.

A noteworthy modification occurs in the treatment of Goursat's theorem that when $f(z)$ has a derivative at every point of a region, this derivative must be continuous. In the first edition Goursat's original proof by the integral method was given, and this was followed by Moore's proof by the differential method. These are now replaced by a beautiful demonstration of the author's own construction (page 349), based on an extension of the theorem of Morera that a function $f(z)$ which is continuous within a given region will be analytic if its integral taken around an arbitrary closed curve within the region is always zero. This extension, which is found only in the new edition, was suggested to Osgood by a similar idea of Bôcher in the treatment of logarithmic potential. It replaces the requirement that $\int f(z)dz$ shall vanish for a contour of arbitrary shape by the less restrictive requirement that it shall vanish when taken around rectangles in the region whose sides are parallel to the real and imaginary axes. This change is an important one for simplicity of application. The theorem occupies a rather prominent place in Osgood's development of the function theory. By its aid Weierstrass' theorem concerning the analytic character of the sum of a uniformly convergent series of analytic functions is established, and likewise the unlimited differentiability of the series term by term.

As specimen minor additions in the new edition I may cite a new and desirable section (II 8, § 7) relating to the six anharmonic ratios and the corresponding division of the com-

plex plane; a brief treatment (II 12, § 8) of the functional equation

$$f(x+y) = \frac{f(x)+f(y)}{1-f(x)f(y)}$$

for $\tan x$; Painlevé's theorem (page 53) on the continuity of the boundary values of a function continuous within a region S; and Osgood's illuminating example of non-uniform convergence (pages 87–89). The section on the convergence of infinite products of functions (II 11, § 7) has been much modified, Bôcher's definition of uniform convergence being now replaced by the more usual Weierstrassian definition.

I turn now to the consideration of the new paragraphs on the uniformization of analytic functions. This subject has naturally a special attraction for the author, not only on account of its intrinsic importance but also because it is at bottom a question of conformal representation, to whose solution he had already made a most vital contribution. The first edition closed with a brief paragraph upon the uniformization of analytic functions. Nearly simultaneously with its appearance the almost complete uniformization was effected by Poincaré and Koebe independently. The inclusion of this was the one thing absolutely necessary to bring the last chapter up to date. This has been effected by adding the new matter at the end without material modification of the plan. In consequence, it seems to me rather difficult for the reader to catch the uniting thread of thought which runs through the chapter. His attention is shifted rather rapidly from one thing to another, though at the close the connection becomes clear. Notwithstanding, the chapter is one of the most fascinating in the entire volume. For this reason, as well as to aid anyone who may wish to skim the chapter superficially, I shall analyze its contents further.

To a considerable degree the arrangement is parallel with historical development. The author starts the chapter by presenting Riemann's problem of building a simply connected piece of a plane, with or without continuous and one-to-one correspondence of boundary, conformally upon a closed or open circle respectively. It is announced that first of all the problem is to be considered for a *closed* region with a regular boundary; in other words, for a "region S." But the very first theorem states that by means of a Green's function

the interior T of *any* simply connected piece of a plane whose boundary consists of more than one point can be built conformally upon the interior of a circle. In fact, it is the problem of the conformal representation of *interiors* which is the key note of the new chapter. Next, the construction of a Green's function for a region S is made to depend upon the solution of the boundary value problem, and this is attained by the Murphy-Neumann-Schwarz alternating process. Returning then to the consideration of open regions, we have next Osgood's own great achievement, the proof of the existence of a Green's function for any simply connected piece T of the character specified above. The proof is applicable, no matter how evil the character of the boundary and irrespective of its possession of inaccessible points. A few words here concerning possible complications of the boundary and the extraordinary things which may happen when account is taken of what becomes of the boundary points in the representation, would not only have been illuminating but would have made visible the difficulty of the theorem.

Next follow paragraphs upon the circular triangle with zero-angles (where we consider again the imaging of a closed region) and upon Picard's theorem. Then the consideration of the uniformization of analytic functions is begun with a novel construction of a simply-connected Riemann surface for any algebraic function. This surface has, in general, an infinite number of leaves, though there is a coordinate case of closure ($p = 0$) which is put to one side by the author for subsequent treatment in a "Nachtrag." It is this surface which, as it presently appears, solves the uniformization problem by its conformal* representation upon the whole or a part of the plane. The surface is introduced to the reader in a paragraph headed: "Der algebraische Fall, Uniformisierung vermöge automorpher Funktionen mit Grenzkreis." But automorphic functions with a limit-circle are not introduced till the close of the next paragraph and are used in only one of the three cases possible for the uniformization. The following sections contain several references (pages 721, 723, 731) to a Hauptsatz,† though by inadvertence nothing has been so marked. The current of development is interrupted by

* Conformal except at winding points.

† The first paragraph of page 715, beginning with line 3, meets essentially the requirements of the references.

certain considerations of analysis situs and by the necessity of proving the well known theorem of Hurwitz on the number of roots of an analytic function $\phi(z) = \lim_{n=\infty} \phi(n, z)$ in a given region.

I have already paused unduly on the one place in the entire volume where the author falls below a uniformly high level of execution. Not a little trouble would be saved the reader if some such orientation as the following had been placed early in the chapter instead of being left to emerge at the end. The fundamental geometrical problem underlying the chapter is that of the conformal mapping of a *simply-connected* region of *interior* points upon the whole or a part of the smooth plane. The region may or may not have a frontier and may contain an infinite number of leaves, but the winding points, so far as they are included as interior points, are to be of finite order (cf. Study's presentation). At such points conformality of representation is, of course, renounced. Three cases arise in the mapping according as the region can be referred to (1) the entire plane, (2) the finite plane, and (3) the interior of the unit circle. It is, indeed, by the renunciation of all boundary considerations that the recent advance has been made by Osgood, Koebe, and Poincaré. When the conformal representation is effected for the simply connected interior of the Riemann surface properly constructed for an analytic function $w = f(z)$, the uniformization by pairs of one-valued analytic functions $z = \phi(t)$, $w = \psi(t)$ at once follows as a corollary, three cases being distinguished according as the domain of the variable t is one or another of the three regions given above.

In this uniformization work Osgood had before him the problem of interpreting the fat pages of Koebe and of getting at their kernel. The task is very far from an easy one, and he deserves our hearty thanks for having accomplished it in brief compass and with great penetration and clearness. The desirability of such a presentation was also perceived in another quarter, and shortly after the publication of Osgood's new edition there also appeared an independent but in some ways different treatment at the opening of the second Heft of Study's Vorlesungen über ausgewählte Gegenstände der Geometrie. The reader will find it most interesting to compare the two presentations. Study and Blaschke throughout the Heft employ Osgood's first edition for reference and, in

particular, in their exposition of Koebe assume many of the results contained in its last chapter.

Koebe's Abbildungsatz (Osgood II 14, § 12) should be pointed out as an especially happy hit, for by means of it the difficult case in which there is no Green's function for the region can be controlled, the surface being then built upon the open finite plane (case 2 above). The greater definiteness imparted to this lemma by Osgood should also be remarked (cf. footnote, page 727). The other two cases (Nos. 1 and 3) can be handled with or without the lemma and are discharged by Osgood through older and less artificial methods. The volume closes with a proof of the very fundamental point—which, so far as I know, was neglected by Koebe—that to every analytic function there corresponds a Riemann's surface, not merely im Kleinen but im Grossen; and conversely, under certain restrictions, to every simply connected Riemann surface an analytic function.

II.

It remains yet to consider the characteristics of the work as a whole. In good measure we knew what to expect from Professor Osgood's pen; clearness, exactness, penetration, refined distinctions and proofs, discrimination between the vital and unessential. In such anticipations, we are not disappointed. The work fairly pulsates with these characteristics. They are so apparent that it will be unnecessary to dwell upon them. To these temperamental qualities there is added the fullness of knowledge which began but was by no means completed in the preparation of the section upon the theory of functions in the Encyklopädie der mathematischen Wissenschaften. The influence of this encyclopædic training is to be seen in the diversity and fertility of content of the present volume. To some extent they impede the swing of matter and style. But there is careful ordering and close connection of parts. Indeed, the different paragraphs and sections are so closely knit together as to necessitate an unusual amount of cross referencing. The reader is kept looking both backwards and forwards and must proceed warily. The book has the prime merit that it can not be read without careful thinking.

Few subjects in mathematics offer such opportunity for variety of treatment as the theory of functions. The selection

and disposition of material must depend largely upon the author's aim and his view of the subject. In the present instance we find the anomaly of a work from the Cauchy-Riemann standpoint by a man of Weierstrassian feeling. From remarks in the introduction regarding the use of power series it is clear that the Weierstrassian development does not seem so fundamental because it is tied up to one form of representation. In the author's language "the theorems gain in clearness by striking off the incidental element which enters in by formulation in terms of power series." Of the far greater rapidity of the Riemann-Cauchy method there can be, I think, no question, provided the necessary prerequisites and knowledge can be assumed. Also there is a gain in steeping the beginner in that method which has proved itself the most powerful but which it is more difficult to master as a tool. Never before has the Cauchy-Riemann development been given with such completeness and precision. To a considerable extent preceding expositions have been on a basis of loose analysis. The revision of the calculus and fundamental principles of analysis in recent time necessitates a reworking of the Cauchy-Riemann development. It is easy to imagine the appeal which such a recasting would make to one endowed with a Weierstrassian love for strict and legal proof. Here Osgood has seen his opportunity and thrown himself into the work.

As has been already said, a thorough comprehension of the Cauchy-Riemann basis requires a considerable acquaintance with analysis. It is accordingly presupposed that the student has a well grounded knowledge of the scientific principles of the calculus, not such a knowledge as is derived from a formal first course so often found advisable for the American student to supplement deficiencies of drill in preliminary subjects, but such a grip as may be gained from an extended second course concerned largely with its fundamental concepts. This course should also be accompanied with a good course in mechanics. It is the author's belief that it is desirable to keep the students in mathematics and physics together as long as possible in the study of analysis, that thereby the students of mathematics, on the one hand, shall perceive its close physical relations and applications, while, on the other hand, the students of physics shall secure a sound working knowledge of analysis, which is with them unfortunately altogether too

rare. To achieve this it is highly desirable to carry the physical student into the function theory and to unroll it in close relation to the subject of potential. Such, as I conceive it, is the author's "Glaubensbekenntniss," the platform beneath his text book; and it is a strong one. It explains also the formation of his Abschnitt III, devoted largely to physical applications and the potential theory. It seems not unlikely that the structure and disposition of the material may exert a strong influence on the teaching of the theory of functions in at least our own country.

Incidentally it may be pointed out that another cause which may have contributed in some measure to the formation of Abschnitt III is the author's fondness for conformal representation. Attention to this subject is a conspicuous feature of the book almost from first to last. I know of no place where it is so adequately and broadly handled.

The discussion just given leads us naturally to consider the question how far the work is adapted for introductory use. The answer to this query must depend largely upon *how* the book is used. It is not an Einleitung for private study. A more descriptive and appropriate term is the word Lehrbuch used in the title. The beginner may, I fear, find it difficult to get the perspective. His feeling might be akin to that of a sightseer left alone to take a first view in an enormous art gallery like the Louvre. Masterpieces of design and portraiture are about him on all sides, and in their multitude he feels lost. Furthermore, the beginner rarely has at the start the capacity for distinctions necessary for proper reading of this book. Large omissions will therefore be advisable for him in a first reading. Certain recommendations in this regard are made at the opening of the fifth chapter but other omissions should also be made, such as naturally occur in following a course of lectures. But for collateral use in a lecture course the book is invaluable, and it is an indispensable handbook for every teacher of the function theory.

If a digression is in place here, a plea might be made for a *brief* introductory course in the Weierstrassian theory. The student becomes familiar in calculus with the Taylor's series. He will be quick therefore to see the importance of concepts and results arising in connection with its further study and will then be ready for application of the ideas to other series whose terms are not powers or power elements. At the same

time concreteness of development is never lost. Inasmuch as the Weierstrassian theory requires less preliminary study, it can be used to introduce the student into the theory of functions at an earlier stage. Pedagogically, therefore, it has its advantages. Yet it is unquestionalby a narrow development and hence its study should not be pushed by the beginner too far. To a far less degree than the Cauchy-Riemann method it involves the perpetuation and broadening of the analysis which is begun with the calculus. Hence it seems not unlikely that an introduction from the other standpoint may become increasingly important and prevalent.

Before leaving general considerations I should not fail to reemphasize one aim and achievement. It is, I think, the *crowning* excellence of the work to have culled out of the heart of the analytic function theory the most vital methods and results and to have given a systematic development of the whole upon a secure foundation of analysis. How great a progress has been made may be seen by comparing the work with some of its predecessors.

I turn now to more detailed considerations. The theorems of Abschnitt I may be divided roughly into two classes according to their elementary or their advanced character. Those of the former class relate mainly to fundamental concepts of the function theories and are particularly well done. The other group concerns the arithmetization of analysis situs and is more exacting in the penetration required. It is the latter to which the specialist will turn and which the author has treated con amore.

As typical of this second group take the familiar theorem that a closed Jordan curve divides the plane into two distinct regions. Intuitionally the theorem is declared obvious, but mathematical progress has shown that there are curves and curves of astounding complexity, even possessing properties contradicting intuition. In Osgood's treatise the student is not allowed to jog along whistling, happy in the belief that the concept curve is something we know all about or is an irreducible element of thought too perfect for analysis. Osgood nowhere tells us what he thinks should be called a curve (we regret the omission), but Jordan curves, simple (i. e., "einfache") curves, analytic curves, regular curves are carefully defined. The above theorem is not proved for any closed Jordan curve but is established by the Ames method for any

closed curve both simple and regular. It is, indeed, the regular curve, composed of a finite number of pieces with continuously turning tangent, which he chooses as the fundamental contour in deriving the integral theorems of Cauchy and others. The statement of these theorems with definite and reliable contour limitations is satisfying. As a characteristic example of what I mean I may refer the reader to the formulation of the familiar theorem (page 333) connecting

$$\frac{1}{2i\pi}\int\frac{f'(z)}{f(z)}dz$$

with the number of roots and number of poles of $f(z)$ in a given region.

A second conspicuous instance of the arithmetization of geometrical theorems is to be found in the treatment of regional theorems; in particular, the theorem that the integral $\int(Pdx+Qdy)$ taken around a contour vanishes if at every interior point of the included region we have $\partial P/\partial y = \partial Q/\partial x$, proper continuity conditions being imposed on P and Q and their derivatives. This result is first obtained on the basis of "Naïve Anschauung" and with the help of Green's lemma for transforming double integrals $\int\int\frac{\partial P}{\partial y}\,dxdy$, etc., into contour integrals. A brief critique of the method (page 132) then follows, and finally a strict proof involving the decomposition (cf. § 9, 10 of chapter 5) of a region with regular contour into a finite number of regions of certain standard forms, termed by the author σ-regions. In this we have some of the author's own work. Even as the Weierstrass curve without a tangent is beyond intuition, so also the shapes of regions may transcend the power of visualization. Hence theorems involving regional consideration require careful arithmetization to obtain an irreproachable basis. Corresponding care is taken in the dissection of a region to obtain the familiar theorems of connectivity (pages 172–176), the loop cuts and cross cuts being here restricted in generality. Like the closed contour in the Jordan theorem, the loop cuts are also assumed to be simple and regular, and the cross cuts are also regular except possibly at their end points. Attentive reading and careful thought are required in these sections of the work. In the new edition there is a noteworthy simplification in the treatment by means of the σ-regions.

Certain features in the first mentioned class of more elementary theorems also deserve mention; for example, the very simple and natural proof after Dini of the fundamental theorem on implicit functions, and the remarkably fine section giving the necessary and sufficient condition for conformal representation, a topic which is only too often slurred over in unsatisfactory fashion. A protest may well be made against the author's use of the term "infinite derivative" (pages 22, 26) to signify one which is either positively infinite or negatively infinite. This would debar the application of the term at a vertical cusp if the curve of the function lies on both sides of the cuspidal tangent.

The analytic function is first introduced in the opening chapter of Abschnitt II, together with its inverse. Then follows a detailed study of the elementary functions and the corresponding conformal representations. The general theory begins with the succeeding chapter, which is devoted to the consideration of functions which are single-valued in a given domain. The structure of this chapter and, in particular, the ordering of the theorems will be of especial interest to the reader. The development is rapid and clear. Here, as elsewhere, the author avoids the ϵ-methods which are of so little interest to the physicist, and in their stead, without sacrifice of rigor, he makes his proofs dependent upon fundamental principles and well-ordered results. Note also the excellent and careful formulation of the contour integration theorems throughout the chapter. Thus, for example, in the very first of these we have the pivotal theorem that an analytic function of z is defined by $\int \phi(t)dt/(t-z)$. Here the contour of integration, though restricted to be simple and regular, is allowed to be open as well as closed, while $\phi(t)$ is any function, real or complex, which is merely supposed to be continuous along the curve. The theorem is also typical of the special attention to parametric integrals, a topic of importance largely neglected in other works on the theory of functions.

The point at infinity is introduced surprisingly late (page 322). But after its introduction the rôle of this "uneigentlicher Punkt" is most carefully observed in theorem and proof. One amusing consequence of its late introduction is to be seen on page 285, where a footnote of the previous edition has been incorporated into the text, thereby making a theorem refer to the point at infinity before its introduction.

Chapter 9, on analytic continuation, is noteworthy on account of the generality and delicacy of its considerations. Painlevé's theorem for continuation (page 434) has here the commanding position, the Weierstrassian method of continuation by means of overlapping circles being given a subordinate place. The chapter closes with a commendable paragraph on the principle of permanence of a functional equation. A nice application is made to the analytic continuation of the Γ-function when defined in the usual manner for the right half-plane by means of an integral, also to the continuation of elliptic functions. With regard to the Weierstrassian treatment of the theory of functions on the basis of power series, it may be said that the sections relating to it are necessarily scattered, so that a view of the method as an organized whole and in its full strength is not directly afforded. Also it does not fall within the scope of the volume to consider in any way a large group of investigations which in one form or another rest on the determination of an analytic function by the coefficients of a power series.

The term "element" of an analytic function is not employed in the narrower Weierstrassian sense of a convergent Taylor's series but is extended to apply to all expressions of form

$$\phi(z) = \sum_{k=m}^{+\infty} c_k (z-a)^{k/n},$$

where n is a positive integer and m any integer positive, negative, or zero. The smooth or winding circular domain of an element may therefore have a pole or a branch point of finite order for its center. The wisdom and even necessity of this broader use becomes luminous in the last chapter, where the inclusion of such elements is indispensable for the uniformization of an analytic function. The truth is, as the author so clearly realizes, that the nature of the element must be adapted to the problem in hand. For many purposes a meromorphic element is obviously more useful than a holomorphic one. As for the Taylor's series with its system of overlapping circles, it is at best adapted to considerations involving a plane with one point lacking, i. e., the point at infinity.

The first two chapters of Abschnitt III gives a succinct and rapid development of fundamental properties of simply as well as doublyperiodic functions. In the succeeding

chapter the elementary functions are considered from an individualistic standpoint unusual for such a treatise. Though the chapter belongs more strictly to the function theory of the real variable, it is nevertheless a welcome interruption, since it is both profitable and stimulating to see old functional friends in new lights. Divers definitions of the functions for real values of the arguments are given. Some of these conduct to characteristic functional equations, for which, conversely, the general solutions are then sought. It will be noted that the first of Cauchy's four fundamental types of functional equations,

$$f(x+y) = f(x) + f(y),$$

is relegated to an exercise, a subordinate position which is altogether natural because of the mode of approach to it but which is totally out of relation to its fundamental importance. The chapter closes with a novel and interesting method applicable in the *real* domain to the derivation of standard expansions for $\sin x$, $\operatorname{ctn} x$, etc., into infinite series and products which make visible their zeros and poles respectively.

It remains yet to speak of the chapter on logarithmic potential. In this we find a development of the theory of logarithmic potential in a series of theorems closely parallel to those of the function theory but on an independent basis. As is so well known, in addition to the Weierstrassian and Cauchy-Riemann bases for the function theory there is also a third one, which rests on the potential theory and follows on separating potential functions into conjugate pairs. I know of no place where this is so completely and delightfully presented as in Osgood's Lehrbuch. It has for us the further interest of being one to which Bôcher has contributed. Many of the theorems, as for instance that for continuation of a harmonic function across a boundary (III 13, § 6), must interest the physicist. I may also call attention to the theorem (page 673) that for a simply connected region the curve along which a Green's function keeps a constant value is a simple and regular closed curve without corners.

The work is distinguished from most or all other works on the theory of functions by a good supply of illustrative exercises. There is also a fine index both of names and matter which contributes greatly to its usefulness.

The system of cross referencing is fatiguing. Beside chapter and paragraph numbers (both in same type) there is often

a Hauptsatz or Satz number to be looked for under the paragraph, and sometimes a Zusatz under the Satz. Inasmuch as the number of cross references is extremely large, the method employed becomes a matter of some importance to the reader. If page references are deemed impracticable because of trouble with the proof and because of the necessity of change with every new edition, it may be suggested that a system preferable to the author's would be a division of the subject matter into short paragraphs not more than a page or two in length and numbered consecutively to the end, as is done in so many French works. The terms and definitions are well chosen and, with rare exceptions, well explained. So far as I can find, the terms Randpunkt for a region and singular point for a function are left undefined, being regarded as self-explanatory or sufficiently clear from the examples given. It is to be regretted that a term so distinctive as "meromorphic" and so universally used is nowhere introduced.

In conclusion, admiration must be expressed for the combination of a grasp "im Grossen" with one "im Kleinen," to apply descriptive terms which are employed by the author and which, if we mistake not, are of his own mint. The appearance of the second volume will be awaited with great interest. Analytic functions of two or more variables will doubtless be considered? What else? From the abundance of material on every hand we may confidently expect to have again a selection of great individuality and interest.

EDWARD B. VAN VLECK.

PARIS,
March 27, 1914.

NOTES.

IN response to the invitation of Brown University to participate in the celebration of its one hundred and fiftieth anniversary, the twenty-first summer meeting of the American Mathematical Society will be held at that university on Tuesday and Wednesday, September 8–9. Titles and abstracts of papers intended for presentation at the summer meeting should be in the hands of the Secretary by August 22.

THE closing (June) number of volume 15 of the *Annals of Mathematics* contains the following papers: "Some solutions

of the Pellian equations $x^2 - Ay^2 = \pm 4$," by E. E. WHITFORD; "On the rank of a matrix," by W. H. METZLER; "On the expression of certain minors of the lth compound of a determinant as a function of the elements of a single line of mth compound," by W. H. METZLER; "Implicit functions at a boundary point," by L. S. DEDERICK; "A formula in the theory of surfaces," by R. D. BEETLE; "The deviations of falling bodies," by F. R. MOULTON; "Properties of certain homogeneous linear substitutions," by H. HILTON.

AT the meeting of the London mathematical society held on April 23 the following papers were read: By P. A. MACMAHON, "A modified form of pure reciprocants possessing the property that the algebraic sum of the coefficients is zero"; by P. A. MACMAHON, "Lattice and prime lattice permutations."

THE Deutsche Mathematiker-Vereinigung will hold its summer meeting at Hanover, September 20–26, in affiliation with the eighty-sixth annual convention of the association of German naturalists and physicians.

THE French association for the advancement of science will hold its next meeting at Havre, July 27–August 2, under the presidency of Professor A. GAUTIER. The British association will participate in the same meeting.

THE Swiss mathematical society will hold its annual meeting at Bern September 2, during the session of the Helvetian society of naturalists.

THE following courses in mathematics are announced for the academic year 1914–1915:

JOHNS HOPKINS UNIVERSITY.—By Professor F. MORLEY: Higher geometry, three hours.—By Professor A. B. COBLE: Modular functions, two hours; Theory of probability, two hours, second half year.—By Dr. A. COHEN: Calculus of variations, two hours.—By Dr. H. BATEMAN: Differential equations of physics, two hours.

UNIVERSITY OF ILLINOIS.—By Professor E. J. TOWNSEND: Functions of a complex variable, three hours; Ordinary and

partial differential equations and advanced calculus, three hours.—By Professor G. A. MILLER: Elementary groups, three hours; Theory of equations and determinants, three hours, second semester.—By Professor H. L. RIETZ: Actuarial theory, three hours, first semester; Averages and the mathematics of investment, three hours, second semester.—By Professor M. FRÉCHET: General analysis, (*a*) abstract sets, two hours; (*b*) functional operations, two hours.—By Professor C. H. SISAM: Algebraic surfaces, three hours; Solid analytic geometry, three hours, second semester.—By Professor J. B. SHAW: General algebra, three hours.—By Professor A. EMCH: Projective geometry, three hours.—By Dr. A. R. CRATHORNE: Calculus of variations, three hours.—By Dr. R. L. BÖRGER: Modern algebra, three hours.—By Dr. E. B. LYTLE: History of mathematics, two hours, second semester; Teacher's course, two hours, first semester.

THE following advanced courses in mathematics are being given at the Belgian universities during the present semester. The list does not include the courses of the first two years nor those offered in the technical schools connected with the universities.

UNIVERSITY OF BRUSSELS.—By Professor E. BRAND: Elliptic functions, two hours; History of mathematics, two hours.—By Professor L. ANSPACH: Dynamics, two hours.—By Professor A. MINEUR: Higher geometry, two hours.

UNIVERSITY OF GHENT.—By Professor A. DEMOULIN: Differential geometry, one hour; Analytic and elliptic functions, two hours.—By Professor M. STUYVAERT: Theory of numbers, one hour; Theory of elimination, one hour.—By Dr. C. SERVAIS: Algebraic curves and surfaces, two hours.

UNIVERSITY OF LIÈGE.—By Professor J. DERUYTS: Elliptic functions, three hours; Partial differential equations and the theory of surfaces, two hours.—By Professor C. LE PAIGE: Spherical astronomy, two hours; Probabilities, one hour; History of mathematics, one hour; Celestial mechanics, two hours.—By Dr. J. FAIRON: Analytic geometry, two hours.

UNIVERSITY OF LOUVAIN.—By Professor C. DE LA VALLÉE-POUSSIN: Partial differential equations of mechanics, two

hours.—By Professor E. PASQUIER: Dynamics, two hours.—By Professor S. DEMANET: Thermodynamics, two hours.—By Dr. G. VERRIEST: Binary forms, one hour.—By Dr. E. GOEDSEELS: Astronomy, two hours; Probabilities, one hour.

MR. G. P. THOMSON, of Trinity College, Cambridge, has been appointed lecturer in mathematics at Corpus Christi College.

PROFESSOR W. F. OSGOOD, of Harvard University, has been appointed Perkins professor of mathematics; Professor W. C. SABINE has been appointed Hollis professor of mathematics and natural philosophy.

PROFESSOR C. J. KEYSER, of Columbia University, received the honorary degree of doctor of laws from the University of Missouri on the occasion of the seventy-fifth anniversary of the foundation of the university.

AT Purdue University Professor WILLIAM MARSHALL has been granted a leave of absence to study a year in France. Mr. R. B. STONE has been promoted to an assistant professorship of mathematics. Dr. T. E. MASON and Mr. O. W. ALBERT have been appointed to instructorships in mathematics.

AT Dartmouth College Drs. J. E. ROWE and E. S. ALLEN have resigned, the latter to accept an instructorship in Brown University. Dr. R. D. BEETLE, of Princeton University, and Dr. L. C. MATHEWSON, of the University of Illinois, have been appointed instructors in mathematics. Professor J. W. YOUNG has accepted the position of chief examiner in geometry of the College entrance board.

PROFESSOR T. M. PUTNAM, of the University of California, will be abroad on leave of absence during the first half of the coming academic year.

DR. W. F. SHENTON has been appointed instructor in mathematics in Johns Hopkins University.

DR. HELEN TAPPAN has been appointed instructor in mathematics at the Iowa State College.

DR. E. A. T. KIRCHER has been appointed instructor in mathematics at the Massachusetts Institute of Technology.

PROFESSOR JULES MOLK, of the University of Nancy, director of the French edition of the Encyclopedia of Mathematical Sciences, died May 7 at the age of 56 years.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

BAER (W. S.). Beiträge zum Waringschen Problem. (Diss.) Göttingen, 1913.

BERANCK (A.). Zur sphärischen Abbildung der Flächen zweiter Ordnung und ihrer stereographischen Projektion. (Progr.) Baden bei Wien, 1913. 8vo. 12 pp.

BERECHNUNG der Zahl π auf Grundlage zyklischer und parabolischer Gesetze. Aussig, 1913.

BOREL (É.). Le Hasard. Paris, Alcan, 1914. 12mo. 4+312 pp. Fr. 3.50

CORRAL (J. I. DEL). Nuevos métodos para resolver ecuaciones numéricas. Madrid, Romo, 1912. 8vo. 22+304 pp. P. 7.00

DOLBNIA (J.). Oeuvres mathématiques. Publiées sous les auspices de l'Ecole supérieure des Mines de l'Impératrice Cathérine II, a Saint-Pétersbourg. Avec une préface de G. Darboux et une notice sur la vie et les travaux scientifiques de Dolbnia, par N. Kryloff. Paris, Hermann, 1913. 8vo. 14+348 pp.

EICHHORN (E.). Konstruktive Ueberführung projektiver Grundgebilde. (Progr.) München, 1913. 8vo. 44 pp.

EXNER (F. M.). Ueber die Korrelationsmethode. (Diss.) Jena, 1913. 8vo. 36 pp. M. 1.00

FALK. Ueber eine symmetrische Darstellung einiger in der Theorie der elliptischen Funktionen vorkommenden Wurzelgrössen. 8vo. M. 3.50

GAUSS (C. F.). Die vier Beweise für die Zerlegung ganzer algebraischer Funktionen in reelle Faktoren 1ten und 2ten Grades. (1799–1849.) Herausgegeben von E. Netto. (Ostwald's Klassiker der exacten Wissenschaften. Nr. 14.) 3te Auflage. Leipzig, Engelmann, 1913. 8vo. 82 pp. Cloth. M. 1.20

GENELIN (S.). Die vier Grundoperationen mit abgekürzten Dezimalzahlen. (Progr.) Krems, 1913. 8vo. 43 pp.

GREBE (F.). Verwendung des Imaginären in der Geometrie. (Progr.) Naumburg a. S., 1913. 8vo. 26 pp.

HABENICHT (B.). Funktionen mit ganzzahligen Hauptpunkten. Linden-Hannover, Ellermann, 1913. 8vo. 9 pp.

HEINE (H.). Arithmetische und geometrische Reihen. Leipzig, Ehlermann, 1913. M. 1.20

JANS (C. DE). Over middelvlakken en middelkrummen. Gand, Hoste, 1913.

KANDA (A.). Ueber rationale Kurvenpunkte. (Progr.) Wien, 1913. 8vo. 10 pp.

KRYLOFF (N.). See DOLBNIA (J.).

LOTS (A.). Beweis des grossen Fermatschen Satzes. Altenburg, Selbstverlag, 1913.

MOORS (B. P.). Etude sur les formules (spécialement de Gauss) servant à calculer des valeurs approximatives d'une intégrale définie. (Koninklijke Akademie van Wetenschappen te Amsterdam.) Amsterdam, J. Müller, 1913. 8vo. 43 pp.

MÜHLE. Ein Beitrag zur Lehre von den pythagoreischen Zahlen. (Progr.) Wollstein i. Posen, 1913. 8vo. 19 pp.

MULLEMEISTER (H.). Over de configuratie $(8_4, 8_4)$ van Punten en Vlakken en de Tetraëders van Moebius. (Diss.) Utrecht, 1913. 8vo. 93 pp.

NETTO (E.). See GAUSS (C. F.).

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